Topological Superconductivity (@ 37 mins)

BdG formalism

$$\mathcal{H} = \left(\frac{p^2}{2m} - \mu\right)_{2\times 2} = \sum_{p\sigma}\left(\frac{p^2}{2m} - \mu\right) c^\dagger_{p\sigma} c_{p\sigma}$$

$$|\Omega\rangle = \prod_{\varepsilon(p)<0} \prod_{\sigma} c^\dagger_{p\sigma} |0\rangle$$

$$\mathcal{H} = \frac{1}{2}\left[\sum_{p\sigma}\left(\frac{p^2}{2m} - \mu\right) c^\dagger_{p\sigma} c_{p\sigma} - \sum_{p\sigma}\left(\frac{p^2}{2m} - \mu\right) c_{p\sigma} c^\dagger_{p\sigma}\right] + \frac{1}{2}\sum_{p\sigma} \varepsilon(p)$$

$$= \frac{1}{2}\sum_{p\sigma}\left[\varepsilon(p)\, c^\dagger_{p\sigma} c_{p\sigma} - (\varepsilon(-p))\, c_{p\sigma} c^\dagger_{p\sigma}\right] + \frac{1}{2}\sum_{p} E(p)$$

$$= \frac{1}{2}\sum_{p\sigma}\left[E(p)\, c^\dagger_{p\sigma} c_{p\sigma} - E(-p)\, c^\dagger_{-p\sigma} c_{-p\sigma}\right] + \frac{1}{2}\sum_{p\sigma} E(p)$$

now use a spinor

$$\psi_p = \left(c_{p\uparrow}\ c_{p\downarrow}\ c^\dagger_{-p\uparrow}\ c^\dagger_{-p\downarrow}\right)$$

$$\mathcal{H} = \sum_p \psi_p^\dagger\, \mathcal{H}_{BdG}(p)\, \psi_p$$

e like spectrum

$$\mathcal{H}_{BdG} = \frac{1}{2}\begin{pmatrix} E(p) & & 0 & \\ & E(p) & & \\ 0 & & -E(-p) & \\ & & & -E(-p) \end{pmatrix}$$

hole like spectrum

four eigenvalues $\rightarrow$ two $E(p)$ &
two $-E(-p)$

Interactⁿ :- $H_{pairing} = \sum_p \Delta C^{+}_{p\uparrow} C^{+}_{-p\downarrow} + \Delta^* C_{-p\downarrow} C_{p\uparrow}$

$$= \sum_p \frac{1}{2} \Delta \left[C^{+}_{p\uparrow} C^{+}_{-p\downarrow} - C^{+}_{-p\downarrow} C^{+}_{p\uparrow} \right]$$

$$+ \frac{\Delta^*}{2} \left[C_{-p\downarrow} C_{p\uparrow} - C_{p\uparrow} C_{-p\downarrow} \right]$$

$$\mathcal{H}_{BdG}(p,\Delta) = \frac{1}{2} \times \begin{pmatrix} \mathcal{E}(p) & 0 & 0 & \Delta \\ 0 & \mathcal{E}(p) & -\Delta & 0 \\ 0 & -\Delta^* & -\mathcal{E}(-p) & 0 \\ \Delta^* & 0 & 0 & -\mathcal{E}(-p) \end{pmatrix}$$

idea of TSC $\rightarrow$ Quasiparticles have a Gap & hence likely that at the edges, there could be topological excitations

(why do I need an insulating bulk again?)

s-wave pairing $\rightarrow$ not very interesting, non-topological character

P-wave : $\Delta = \Delta(P_x + iP_y)$ $\rightarrow$ can lead to topological property

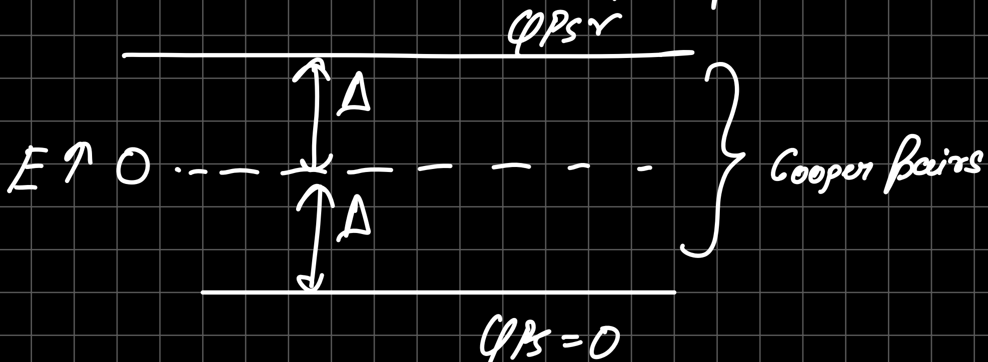
Starts looking like a dirac eqn
(not a s-wave situatⁿ)

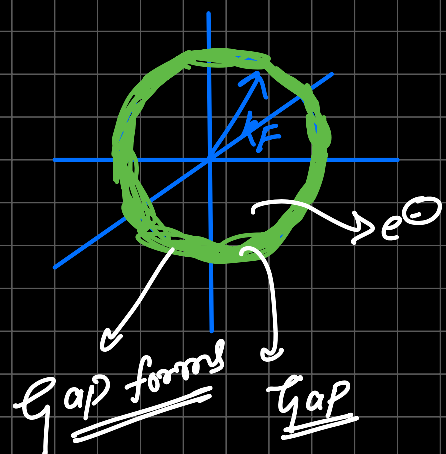
@ Bernevig's book & Shen's book

BCS

$$H_{eff} = \sum_k \left\{ \xi_k c_k^+ c_k + \frac{1}{2}\left(\Delta_k^* c_{-k\downarrow} c_{k\uparrow} + \Delta_k c_{k\uparrow}^+ c_{-k\downarrow}^+ \right) \right\}$$

k + $-k$ form cooper pairs.





A SC gap isn't the same as an insulator Gap but it is roughly the same

Cooper pair numbers aren't conserved.

$$\xi = \frac{k^2}{2m} - \mu$$

interesting physics @

$$\Delta_k = \Delta(k_x + i k_y) \quad \{ l = 1$$

$$\Delta_k = \Delta(k_x - i k_y) \quad \{ l = 1$$

Sr_2RuO_4 → expected "p-wave"

but for He^4, p-wave is accepted.

Superfluidity

He^3 → formionic

→ behaves as a superfluid, analogous to pairing in a superconductor

&

"proved to be p-wave"

∴ while in actual SC p-wave pairing candidates are few, in other systems it's present.

Emergent p-wave → possible to engineer a system to manifest p-wave pairing